# Statistics

# Lecture 13

Feb 19-8:47 AM

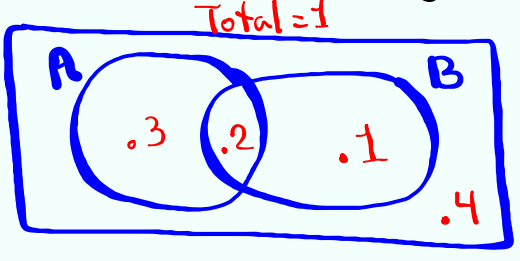
Class Quiz 5

Given $P(A) = .5$, $P(B) = .3$, $P(A \text{ and } B) = .2$

$.5 - .2 = .3$

$.3 - .2 = .1$

1) Construct Venn Diagram



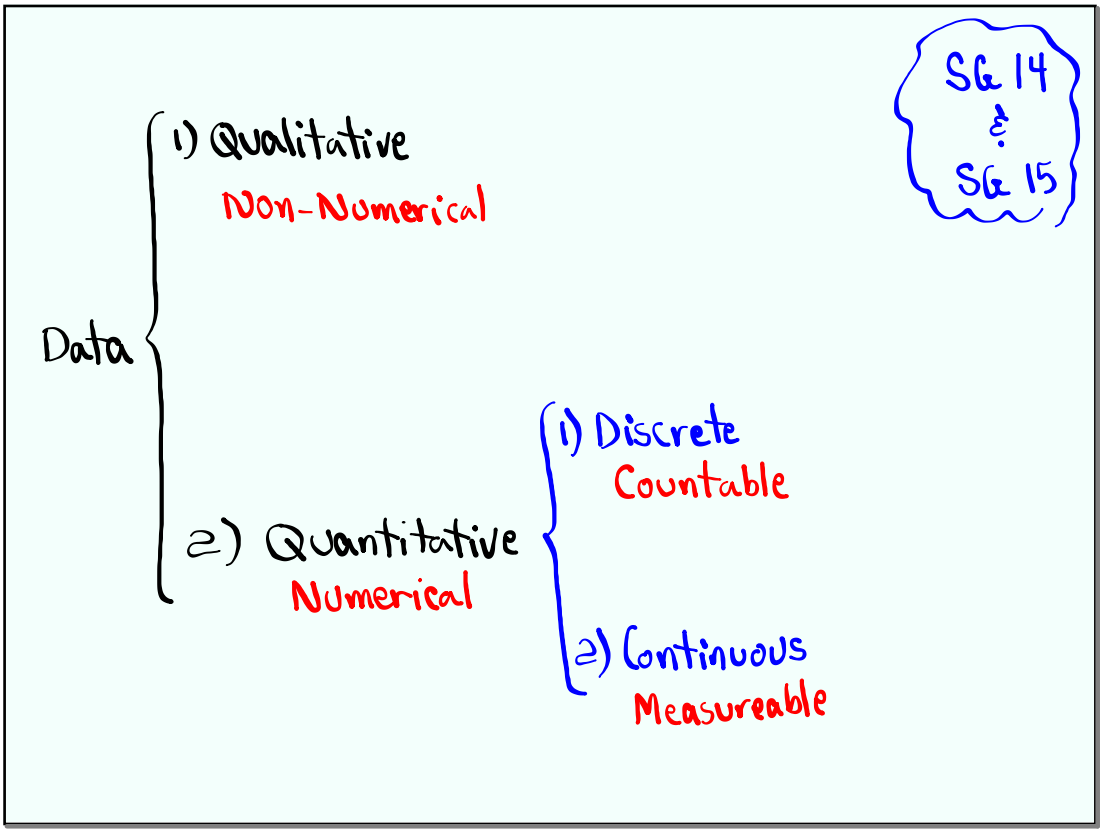

2) Find $P(B|A)$

Given

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.2}{.5}$$

$$= \frac{2}{5} = .4$$

Mar 26-1:45 PM

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Discrete Random Variable

Let $x$ be a discrete random variable with Prob. dist. $P(x)$.

Prob. dist. provides prob. of all possible outcomes.

Prob. dist. can be in the form of a table, graph, some formula, or we simply find prob. of all outcomes.

1) $0 \le P(x) \le 1$
2) $P(x) = 1 \iff$ Sure event
3) $P(x) = 0 \iff$ Impossible event
4) $0 < P(x) \le .05 \iff$ Rare event
5) $\sum P(x) = 1$

Mar 26-1:57 PM

Consider the chart below

| x | P(x) |
|---|---|
| 1 | .2 |
| 2 | .5 |
| 3 | .3 |

1) Verify $\sum P(x) = 1$ ✓

$.2 + .5 + .3 = 1$ ✓

2) Find $P(x=2 \text{ or } x=3)$

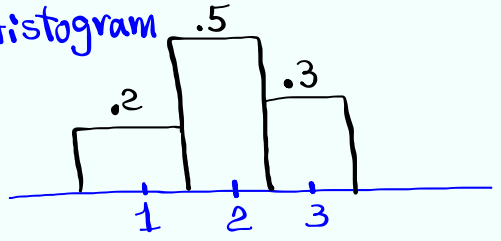
$= P(x=2) + P(x=3)$

$= .5 + .3 = \boxed{.8}$

3) Draw Prob. dist. Histogram

$x \rightarrow$ Midpoint

$P(x) \rightarrow$ Rel. F.




Mar 26-2:03 PM

Consider the chart below

| x | P(x) |
|---|---|
| 1 | .1 |
| 2 | .3 |
| 3 | .4 |
| 4 | .2 |

1) Find $P(x=4)$

$P(x=4) = 1 - [.1 + .3 + .4]$

$= 1 - .8 = \boxed{.2}$

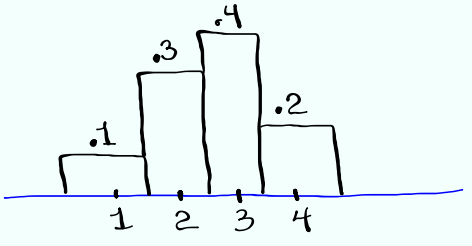
Total Prob. = 1

2) $P(x \geq 2) = .3 + .4 + .2 = \boxed{.9}$

3) Draw Prob. dist. histogram.

$x \rightarrow$ Midpoint

$P(x) \rightarrow$ Rel. F.




Mar 26-2:07 PM

2 Red 3 Blue Balls

Take 2 Balls **No replacement**

RR RB BR BB Sample Space

Let $x$ be # of Red balls in each outcome

RR $\rightarrow x=2$ $P(x=2)=\frac{2}{5}\cdot\frac{1}{4}=\frac{1}{10}=.1$

RB $\rightarrow x=1$ $P(x=1)=.6$

BR

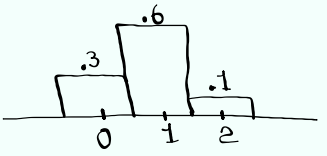
BB $\rightarrow x=0$ $P(x=0)=\frac{3}{5}\cdot\frac{2}{4}=\frac{3}{10}=.3$

| $x$ | $P(x)$ |
|---|---|
| 2 | .1 |
| 1 | .6 |
| 0 | .3 |

P(Both balls are Same Color)

= P(RR or BB) = .1 + .3 = **.4**

Draw Prob. dist. histogram




Mar 26-2:13 PM

Complete the chart below

| $x$ | $P(x)$ | $xP(x)$ | $x^2P(x)$ |
|---|---|---|---|
| 0 | .3 | 0 | 0 |
| 1 | .6 | .6 | .6 |
| 2 | .1 | .2 | .4 |

1) $\sum P(x)=1$

2) $\sum xp(x)=.8$

3) $\sum x^2p(x)=1$

4) $\sum x^2p(x)-\left(\sum xp(x)\right)^2=1-.8^2=1-.64=$ **.36**

5) $\sqrt{\text{Last Answer}}=\sqrt{.36}=$ **.6**

$x \rightarrow L1$

$P(x) \rightarrow L2$

| L1 | L2 |
|---|---|
| 0 | .3 |
| 1 | .6 |
| 2 | .1 |

STAT → CALC

1:1-Var Stats

List: L1

Freq list: L2

Calculate

L1, L2

Enter

$\bar{x}=.8$

$S_x=$ Blank

$n=1$ ← Total Prob.

Mar 26-2:21 PM

Mean $\mu$ MU

Variance $\sigma^2$ Sigma$^2$

Standard deviation $\sigma$ Sigma

$\mu = \sum x\,p(x)$

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$\sigma = \sqrt{\sigma^2}$

| $x$ | $P(x)$ |
|---|---|
| 1 | .15 |
| 2 | .25 |
| 3 | .45 |
| 4 | .15 |

1) $\sum P(x) = .15 + .25 + .45 + .15 = 1$ ✓

$x \to$ L1, $P(x) \to$ L2

STAT → CALC

1:1-Var Stats

Use L1 & L2

$\mu = \bar{x} = 2.6$

$\sigma = \sigma_x = .917$

$n = 1$

$\sigma^2$ in reduced fraction

VARS → 5: Statistics → 4: $\sigma_x$

$x^2$ → MATH → 1: ▶Frac → Enter

$\sigma^2 = \frac{21}{25}$

Mar 26-2:32 PM

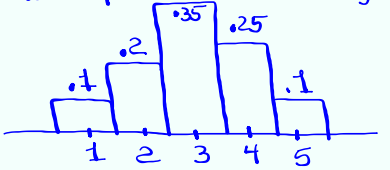
Consider the chart below

| $x$ | $P(x)$ |
|---|---|
| 1 | .1 |
| 2 | .2 |
| 3 | .35 |
| 4 | .25 |
| 5 | .1 |

1) $P(x=5) = 1 - [.1 + .2 + .35 + .25] = .1$

2) $P(2 \le x \le 4) = .2 + .35 + .25 = .8$

3) Draw prob. dist. histogram




$x \to$ L1

$P(x) \to$ L2

1-Var Stats with L1 & L2

4) Find $\mu$ & $\sigma$

$\mu = \bar{x} = 3.05$

$\sigma = \sigma_x = 1.117$

$n = 1$

5) $\sigma^2$ in Reduced fraction

VARS → 5: Statistics → 4: $\sigma_x$

$x^2$ → MATH → 1: ▶Frac → Enter

$\sigma^2 = \frac{499}{400}$

Mar 26-2:42 PM

$\mu \approx 3$

$\sigma \approx 1$

68% Range $\mu \pm \sigma$

$= 3 \pm 1 \Rightarrow$ 2 to 4

95% Range $\mu \pm 2\sigma$

Usual Range $= 3 \pm 2(1) \Rightarrow$ 1 to 5

99.7% Range $\mu \pm 3\sigma \Rightarrow$ 0 to 6

Mar 26-2:51 PM

I am giving away a Calc worth $100.

I sold 20 tickets at $10 each.

One ticket is drawn → Winning TKT

| Net | P(Net) |
|---|---|
| 10 − 100 | 1/20 |
| 10 − 0 | 19/20 |

Net → L1

P(Net) → L2

Expected Value per TKT

$\mu = \bar{x} =$ 5 $

1-Var Stats

with L1 & L2

Mar 26-2:54 PM

Amber is going on vacation.

She buys insurance for her luggage.
She paid $50

Any damages, Airline pays her $1000.

Prob. of damage is .2%.

Find expected value per policy sold.

| Net | P(Net) |
|---|---|
| 50 − 1000 | .2% = .002 |
| 50 − 0 | .998 |

damage
damage

Net → L1

P(Net) → L2

E.V. = $\mu = \bar{x}$ $48

1-Var stats
L1 & L2

Mar 26-3:00 PM

Give me $10, Draw a Card from a standard deck of playing cards.

| If You draw | I give You |
|---|---|
| Ace | $25 |
| face | $10 |
| any other Card | $0 |

| Net | P(Net) |
|---|---|
| 10 − 25 | 4/52 |
| 10 − 10 | 12/52 |
| 10 − 0 | 36/52 |

Ace
face
any other Card

E.V. Per play for the house

$\mu = \bar{x} \approx$ $5.77

Net → L1

P(Net) → L2

1-Var Stats
L1 & L2

Mar 26-3:07 PM

7 Nickels

3 Quarters

Take 2 Coins

With replacement

| Total ¢ | P(Total ¢) |
|---|---|
| 10 | .49 |
| 30 | .42 |
| 50 | .09 |

SG 14 & 15

NN → 10¢ $P(10¢) = \frac{7}{10} \cdot \frac{7}{10} = .49$

NQ
QN → 30¢ P(30¢) = .42

QQ → 50¢ $P(50¢) = \frac{3}{10} \cdot \frac{3}{10} = .09$

Total → L1

P(Total) → L2

$\mu = 22$

$\sigma = 12.961$

~~Reduced fraction~~ $\sigma^2 = 168$

Mar 26-3:13 PM

Binomial Prob. Dist. SG 16

1) $n$ independent events

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

$p + q = 1$

$q = 1 - p$

$p$ & $q$ remain unchanged for all events.

3) $x$ is # of Successes

$n - x$ is # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Mar 26-3:22 PM

A loaded coin is tossed 10 times.

Success is to land tails.

P(Land tails on each toss) = .4

$n = 10$ $p = .4$ $q = .6$

P(land tails exactly 3 times)

$P(x=3) = {}_{10}C_{3} \cdot (.4)^{3} \cdot (.6)^{7}$

$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

10 MATH PRB $_nC_r$ 3 × .4 ^ 3 × .6 ^ 7

Enter .215

Mar 26-3:28 PM

You are taking a multiple-choice quiz.

20 questions True/False only.

You are making random guesses.

$n = 20$ $p = .5$ $q = .5$

P(guess correctly exactly 12 questions)

$P(x=12) = {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^{8} = .120$

$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

TI Command

2nd VARS ↓ binompdf(

Trials: 20

P: .5

x-Value: 12

Paste Enter

No Menu

20, .5, 12

enter

Mar 26-3:35 PM